5. Von Karman Institute for Fluid Dynamics, Education and Research 1956-1976.
6. G. N. Machekhina and N. I. Khvostov, "Wind tunnels for studies at high Reynolds numbers," Obz. Tsentr. Aerogidrodinam. Inst., No. 353 (1971).
7. J. Lucasiewich, in: Experimental Methods of Hypersonics, Vol. 3, New York (1973).
8. A. A. Meshcheryakov, V. I. Pinakov, and M. E. Topchiyan, "Distribution of stresses in a wedged piston and stem of a adiabatic gas compression system," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1980).
9. B. V. Boshenyatov, A. S. Korolev, et al., Shock Tubes in Aerodynamic Studies [in Russian], Nauka, Novosibirsk (1978).
10. G. F. Bugaro (editor), Technique of Hypersonic Studies [Russian translation], Mir, Moscow (1964).
11. Advances in Hypervelocity Techniques, Plenum Press, New York (1962).
12. J. Lucasiewich, "Experimental investigation of hypervelocity flight," in: First Int. Cong. of the Aeronautical Sciences, Madrid (1958).

INFLUENCE OF THE SHAPE OF THE SUPERSONIC PART OF A NOZZLE ON THE RATE OF REDISTRIBUTION OF MOLECULES OVER VIBRATIONAL LEVELS IN THE ACTIVE MEDIUM OF A CO GASDYNAMIC LASER

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In the majority of experimental reports devoted to the study of CO gasdynamic lasers (GDL) [1-5] conical nozzles having expansion half-angles of 7.5-12° in the supersonic part have been used, and in these reports the resonator was at a considerable distance (0.5-1 m) from the critical cross section of the nozzle. In [6, 7], however, the possibility of creating CO GDL with shorter nozzles was determined experimentally (expansion half-angle 26°, distance from nozzle critical cross section to resonator axis 0.065 m). Theoretical investigations made in [8-11] showed that the process of vibrational relaxation in the active medium of a CO gasdynamic laser can be divided into two regions. The most intense deactivation of vibrational energy takes place in the first region (near the nozzle critical cross section at a high temperature and high pressure), while in the second region (at low pressures and temperatures) the vibrational–translational deactivation is slowed, since the process of redistribution of molecules over vibrational levels is primary. To assure the efficient "freezing in" of vibrational energy, the gas must be transferred from the first region to the second in the shortest time, i.e., in the first region the minimum nozzle diameter and the maximum possible expansion angle are required. The question of what the nozzle profile in the second region should be to assure that the maximum value of the optical amplification factor of the active medium is achieved at the minimum distance from the nozzle critical cross section has not been studied. The results of calculated investigations of the process of vibrational relaxation of carbon monoxide molecules in supersonic streams with different expansion geometries are given in the present report.

We consider the steady, quasi-one-dimensional, adiabatic flow of a CO–Ar mixture in a plane supersonic nozzle with a given configuration along the x axis. It is assumed that only one-quantum transitions take part in the relaxation processes and that local equilibrium between the translational and rotational degrees of freedom exists at each point of the stream. Equilibrium over all degrees of freedom is assumed in the nozzle critical cross section. Within the framework of the enumerated assumptions, the equations of kinetics of vibrational relaxation are written in the form

$$u\frac{dC_v}{dx} = K_{v+1,v}C_{v+1}N - (K_{v,v-1} + K_{v,v+1})C_vN + \quad (1)$$

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$$+K_{v-1,v}C_{v-1}N+\left\{\left(\sum_{w=0}^{w^*}K_{v+1,v}^{w,w+1}C_w\right)C_{v+1}\right.$$
$$-\left[\left(\sum_{w=0}^{w^*}K_{v,v-1}^{w,w+1}C_w\right)+\left(\sum_{w=1}^{w^*}K_{w,w-1}^{v,v+1}C_w\right)\right]C_v$$
$$\left.+\left(\sum_{w=1}^{w^*}K_{w,w-1}^{v-1,v}C_w\right)C_{v-1}\right\}+A_{v+1,v}C_{v+1}-A_{v,v-1}C_v,$$

where v = 0, 1, ..., v*; w = 0, 1, ..., w*; u is the velocity of the gas stream; C_v is the population of the v-th vibrational level; $K_{v+1,v}$ and $K_{v+1,v}^{w,w+1}$ are rate constants for vibrational–translational and vibrational–vibrational transitions, respectively; N is the total number of molecules per unit volume; $K_{v+1,v} = (K_{v+1,v}^{CO}\psi CO + K_{v+1,v}^{Ar}\psi_{Ar})$; $K_{v+1,v}^{CO}$ and $K_{v+1,v}^{Ar}$ are rate constants for vibrational–translational deactivation of carbon monoxide molecules in the v + 1-th level in collisions with CO and Ar molecules, respectively; ψCO and ψ_{Ar} are the molar concentrations of carbon monoxide and argon; $A_{v,v-1}$ is Einstein's coefficient for transitions v → v − 1. The system of relaxation equations (1) was solved jointly with the equations of continuity of the medium and of conservation of mementum and energy, with the equation of state of an ideal gas, and with the initial conditions assigned in the nozzle critical cross section. The populations of the vibrational levels and the optical amplification factors of the medium were calculated in the work.

The amplification factor for the vibrational–rotational transitions v, j − 1 → v − 1, j was calculated using the expressions

$$\alpha_{v,j}=\frac{\lambda_{v,v-1}^2A_{v,v-1}[\ln(2)]^{1/2}}{8\pi^{3/2}\Delta\nu_D}\left(C_{v,j-1}-\frac{g_{v,j-1}}{g_{v-1,j}}C_{v-1,j}\right)H[a,0],$$

$$H[a,0]=\frac{a}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-y^2)}{y^2+a^2}dy,\quad a=\frac{\Delta\nu_c}{\Delta\nu_D}(\ln 2)^{1/2},$$

where $\Delta\nu_D=\frac{1}{\lambda_{v,v-1}}\left(\frac{2RT\ln 2}{\mu}\right)^{1/2}$ is the Doppler half-width of the line; $\Delta\nu_c=p\left(\frac{300}{T}\right)^{0.7}\sum_M\delta_{CO-M}(300,j)\psi_M$ is the collisional half-width of the line; p and T are the pressure and translational temperature of the gas; δ_{CO-M} is the half-width of the spectral line due to collisions between CO molecules having a rotational quantum number j and molecules of the M-th type at a temperature of 300°K [12, 13]; $\lambda_{v,v-1}$ is the emission wavelength for a vibrational–rotational transition; $g_{v,j-1} = 2j - 1$, $g_{v-1,j} = 2j + 1$.

With allowance for equilibrium between translational and rotational degrees of freedom, we obtain

$$C_{v,j-1}=g_{v,j-1}C_v\frac{B_v}{T}\exp(-B_v(j-1)j/T),$$

B_v being the characteristic rotational temperature of the v-th level. In the calculations we used the rate constants for vibrational–translational deactivation and vibrational–vibrational exchange with allowance for the recommendations of [10, 14, 15].

The populations of vibrational levels of carbon monoxide molecules in different cross sections of the supersonic stream were calculated by numerical integration of the system (1) together with the equations of conservation of mass, momentum, and energy of an ideal gas. The initial conditions were assigned in the nozzle critical cross section. In the calculations we allowed for 26 vibrational levels v = 0, 1, ..., 25; the populations of levels with numbers v ⩾ 26 were taken as equal to zero. We analyzed gas motion in plane V-shaped nozzles having an expansion half-angle φ = 26.5° and a nozzle critical cross section with a height $h_* = 3\cdot10^{-3}$ m. The expanding section of the nozzle changes into a channel of constant cross section with the following expansion ratios of the gas stream: $S/S_* = 24.9, 40.1, 79.2, 199$. In addition, we made calculations under the assumption of instantaneous expansion of the gas stream. In this case the gas is cooled to a given translational temperature immediately beyond the nozzle critical cross section.

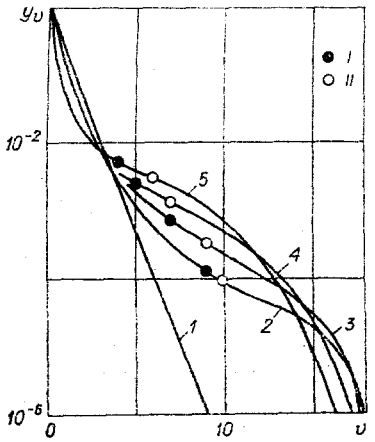

Fig. 1

The calculations made for V-shaped nozzles joined with a channel of constant cross section showed that at one and the same distance from the nozzle critical cross section (with a fixed composition and fixed pressure and temperature in the nozzle critical cross section) the departure of the values of the populations of vibrational levels from the initial distribution is stronger, the lower the temperatures of gas expansion. This statement is valid for vibrational levels with numbers v equal to or several units larger than the number $v_m(x) = [(1 - 2\varepsilon)/\varepsilon T + T1]/(2T1)$ corresponding to the minimum of the Treanor distribution (T1(x) is the vibrational temperature of the first level in the cross section located at a distance x from the nozzle critical cross section and ε is the anharmonism of CO molecules).

The distributions of carbon monoxide molecules over vibrational levels for nozzles having a V-shaped expanding section at the critical cross section (straight line 1) and at a distance of $6 \cdot 10^{-2}$ m from the nozzle critical cross section ($\xi = (x \operatorname{tg} \varphi)/h_* = 100$) (curves 2-5) are presented in Fig. 1 for $p_* = 10$ Pa, $T_* = 2000°K$, a medium with a composition of 0.2 CO + 0.8 Ar, and a relative concentration of molecules $y_v = C_v/N$. The set of expansion ratios of V-shaped nozzles listed above corresponds to translational temperatures of 200, 150, 100, and 57.9°K (curves 2-5, respectively). Points I are vibrational levels, the numbers of which are equal to the value of $v_m(x)$ rounded to integers, and points II are the upper vibrational levels of the transition with the maximum value of the optical amplification factor. As the gas moves in the channel of constant cross section joined to the expanding part of the nozzle there is an increase in the populations of levels with numbers close to v_m and in the optical amplification factor, although these processes proceed more slowly (in a mixture of 0.2 CO + 0.8 Ar) than in an expanding nozzle without a channel of constant cross section.

On the basis of these calculations we can conclude that as the gas moves in the expanding nozzle there is a monotonic shift of the position of the maximum of the amplification factor toward smaller v in the space of vibrational levels. The maximum value of the amplification factor corresponds to transitions $v_m + 2 \rightarrow v_m + 1$ and $v_m + 1 \rightarrow v_m$. This conclusion is confirmed by experimental results [2]. In these experiments with deep cooling of a mixture of 5% CO + 15% N_2 + 80% Ar in an expanding nozzle ($S/S_* = 2730$) the parameter v_m, characterizing the position of the minimum of the Treanor distribution in the space of levels, reached its minimum value ($v_m = 1$). A study of the spectral composition of the emission at the nozzle exit [2] showed that the most intense generation took place on the transition $2 \rightarrow 1$, i.e., on the transition $v_m + 1 \rightarrow v_m$.

The shift of the maximum of the optical amplification factor toward smaller v, due to cooling of the gas during expansion, leads to a decrease in the path of CO molecules in the space of levels from low vibrational levels to the level corresponding to the maximum of the optical amplification factor. At the same time, expansion of the gas is accompanied by a decrease in the number of molecules per unit volume and slowing of the process of transfer of quanta from lower to upper vibrational levels.

To clarify how the rates of change of the populations of vibrational levels depend on the expansion ratio of the gas stream, let us consider gas motion under the conditions of

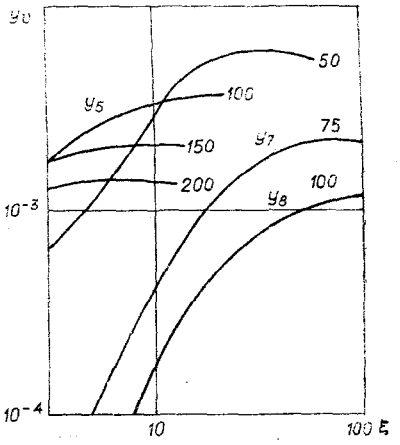

Fig. 2

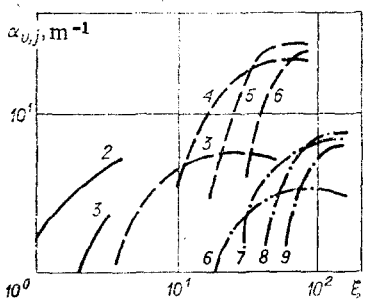

Fig. 3

instantaneous expansion. In this case the values of the translational temperature, pressure, and velocity corresponding to the given expansion ratio are established in the gas immediately after the critical cross section. We considered expansion of the gas to temperatures of 30-200°K. The calculations were made for the following region of variation of the parameters: temperature in the nozzle critical cross section $T_* = 1000$-3000°K, pressure in the nozzle critical cross section $5 \cdot 10^6$-10^7 Pa, carbon monoxide concentration $\psi_{CO} = 0.2$-1.0. The results of the calculations for $T_* = 2000$°K, $p_* = 10^7$ Pa, and $\psi_{CO} = 0.2$ are presented in Figs. 2 and 3. In Fig. 2 the populations of vibrational levels with the numbers 5, 7, and 8 are given in the form of functions of the distance from the nozzle critical cross section; the numbers near the lines correspond to the translational temperatures of the gas. In Fig. 3 the optical amplification factors are given as functions of the distance to the nozzle critical cross section under the conditions of instantaneous expansion of the active medium to translational temperatures of 30, 50, and 100°K (solid, dashed, and dot–dash lines, respectively); the numbers near the curves correspond to the vibrational quantum numbers of the upper levels for the given emission lines. The amplification factors are largest for the vibrational–rotational band.

On the basis of the calculated results it is shown that for a fixed vibrational level the length of the path in the channel of constant cross section over which a quasistationary population of a given level is formed increases with an increase in the expansion ratio of the gas stream. This fact is illustrated in Fig. 2 by the curves characterizing the variation of the populations of the fifth level as a function of the distance to the nozzle critical cross section during expansion of the active medium to translational temperatures of 50-200°K. The increase in the distance required for the population to emerge at the quasistationary value is explained both by the slowing of processes of redistribution of molecules and by an increase in the quasistationary values of the populations of vibrational levels with a decrease in the translational temperature. Since the position of the maximum of the optical amplification factor shifts monotonically toward smaller v with an increase in the expansion ratio of the gas stream, however (see Fig. 3), the distance in the space of levels which molecules travel from lower levels to the levels corresponding to the maximum amplification factor decreases. Such a decrease in the path of molecules for an active medium containing a considerable amount (more than 40%) of argon with expansion in a super-

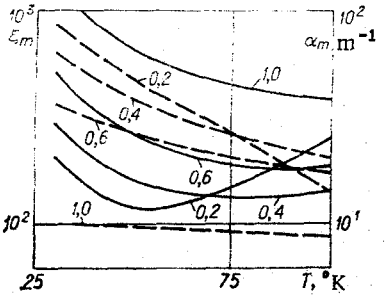

Fig. 4

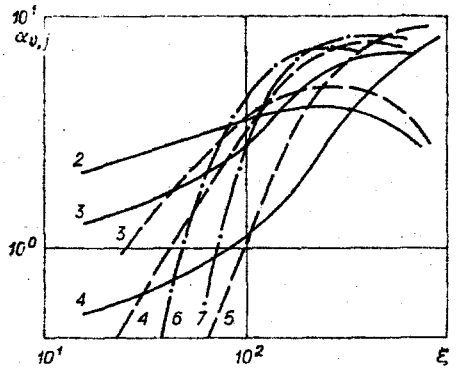

Fig. 5

sonic nozzle from $T_* = 2000°K$ to translational temperatures $T = 50-200°K$ compensates for the slowing of processes of V–V exchange due to the decrease in the gas density and temperature during expansion. The distances needed to reach the maximum amplification factors and the quasistationary populations of vibrational levels corresponding to the maximum amplification factors for the given conditions decrease with an increase in the expansion ratio of the gas stream. The distances ξ_m at which the maximum amplification factors are reached as functions of the translational temperature of the gas in the channel of constant cross section for mixtures with different contents of carbon monoxide are given in Fig. 4 (solid lines) for expansion from $T_* = 3000°K$ and $p_* = 10^7$ Pa (the numbers by the lines correspond to different values of ψ_{CO}). These calculations showed that for each composition of the working medium with a given reserve of vibrational energy there is an expansion ratio of the gas stream at which ξ_m is smallest. A decrease in the argon content in the active medium leads to a shift in the position of the minimum of ξ_m toward smaller expansion ratios of the gas stream, which is explained by the sharper decrease in density in mixtures with a lower argon content upon cooling to the same temperatures.

In the entire investigated region of variation of the composition of the active medium and the expansion ratio of the gas stream the maximum amplification factor of the active medium of a CO GDL grows monotonically with an increase in the expansion ratio of the gas stream. The maximum amplification factors α_m for $T_* = 3000°K$ and $p_* = 10^7$ Pa as a function of the translational temperature of the gas in the channel of constant cross section are given in Fig. 4 (dashed lines); the numbers by the lines correspond to the molar concentrations of carbon monoxide. The increase in the maximum values of the optical amplification factor with a decrease in the temperature of the working medium is determined both by the change in the distribution of molecules over rotational levels and by the shift of the position of the maximum of the amplification factor in the space of levels toward smaller vibrational quantum numbers since, for a given reserve of vibrational energy per unit mass of gas, the smaller $v_m + 1$ (and hence the smaller E_{v_m+1}), the larger the fraction of carbon monoxide molecules which can be concentrated in the level corresponding to the maximum value of the amplification factor.

In CO + Ar mixtures with a carbon monoxide content $\psi_{CO} \leq 0.6$, despite the fact that the parameter ξ_m passes through a minimum in the region of expansion ratios resulting in cooling to temperatures of 30-100°K, the amplification factor in all sections of the channel of constant cross section is higher, the lower the translational temperature of the

gas. The increase in the amplification factor in these mixtures due to a decrease in the translational temperature of the active medium compensates for its decrease due to a decrease in density and due to slowing of the processes of redistribution of carbon monoxide molecules over vibrational levels. Consequently, to provide the highest optical amplification factor at a given distance from the nozzle critical cross section in the active medium of a CO GDL containing more than 40% inert gas one must choose that nozzle profile which provides the fastest possible expansion (as the gasdynamic conditions permit) of the active medium. The expansion ratio of the gas stream must be the highest (expansion ratios providing cooling of the gas stream down to 30°K were considered in our calculations) and is determined by the conditions of the absence of condensation of the working medium.

In mixtures with a low content of inert gas, particularly in pure carbon monoxide, the maximum optical amplification factor increases monotonically as the temperature of the active medium decreases due to the increase in the expansion ratio of the gas stream, although the slowing of processes of V—V exchange due to the decrease in density and temperature leads to the fact that for supersonic nozzles of a given length an increase in the expansion ratio can decrease the amplification factor of the active medium. In Fig. 5 we present the optical amplification factors on different levels (numbers by curves) in pure carbon monoxide as functions of the distance to the nozzle critical cross section for the case of instantaneous expansion ($p_* = 10^7$ Pa, $T_* = 3000$°K, $\xi = x/2h_*$, $\psi_{CO} = 1.0$, T = 30, 50, and 100°K, solid, dashed, and dot—dash lines, respectively). Consequently, to obtain the highest optical amplification factor in the active medium of a CO GDL at a given distance from the nozzle critical cross section with high carbon monoxide concentrations, $\psi_{CO} \sim 1$, the profile of the supersonic part of the nozzle must be chosen with allowance for the kinetics of the redistribution of carbon monoxide molecules over vibrational levels.

LITERATURE CITED

1. W. S. Watt, "Carbon monoxide gas dynamic laser," Appl. Phys. Lett., 18, No. 11 (1971).
2. R. L. McKenzil, "Laser power at 5 μ from the supersonic expansion of carbon monoxide," Appl. Phys. Lett., 17, No. 10 (1970).
3. V. A. Belavin, G. V. Abrosimov, et al., "Coefficient of amplification of a weak signal in a CO gasdynamic laser," Zh. Tekh. Fiz., 47, No. 3 (1977).
4. G. A. Andronov, A. G. Armer, et al., "A gasdynamic laser based on a CO—Ar mixture," Kvantovaya Elektron. (Moscow), 4, No. 8 (1977).
5. B. S. Aleksandrov, G. A. Andronov, et al., "Energy characteristics of the working media of CO gasdynamic lasers," Teplofiz. Vys. Temp., 16, No. 5 (1978).
6. V. F. Gavrikov, A. P. Dronov, et al., "A carbon monoxide gasdynamic laser," Kvantovaya Elektron. (Moscow), 1, No. 1 (1974).
7. V. F. Gavrikov, A. P. Dronov, et al., "An experimental investigation of gasdynamic lasers based on mixtures of CO with inert gases," Kvantovaya Elektron. (Moscow), 2, No. 1 (1975).
8. J. W. Rich, "Kinetic modeling of the high-power carbon monoxide laser," J. Appl. Phys., 42, No. 7 (1971).
9. K. Nanbu, "Vibrational relaxation of anharmonic oscillation in expansion nozzles," J. Phys. Soc. Jpn., 40, No. 5 (1976).
10. V. F. Gavrikov, A. P. Dronov, et al., "Vibrational relaxation of carbon monoxide in supersonic nozzles," Kvantovaya Elektron. (Moscow), 3, No. 7 (1976).
11. N. Ya. Vasilik, V. A. Vakhnenko, et al., "Energy characteristics of a carbon monoxide gasdynamic laser," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1978).
12. D. Williams, D. C. Wenstrand, et al., "Collisional broadening of infrared absorption lines," Mol. Phys., 20, No. 5 (1971).
13. R. H. Hunt, R. A. Tath, and E. K. Plyler, "High-resolution determination of the width of self-broadened lines of carbon monoxide," J. Chem. Phys., 49, No. 9 (1968).
14. N. G. Basov, V. I. Dolinina, et al., "A theoretical investigation of the generation characteristics of a CO electric-ionization laser," Preprint Fiz. Inst. Akad. Nauk SSSR No. 1 (1976).
15. Yu. B. Konev, I. V. Kochetov, et al., "An analysis of the kinetic processes determining the parameters of CO lasers," Preprint Inst. At. Energ. No. 2821 (1977).